

# An Improved Quantum Crossover Operator for Binary Evolutionary Optimization of Thinned Array Antennas

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**Abstract**— Many engineering optimization problems may be rephrased in terms of equivalent binary problems, and these can be effectively tackled with Evolutionary Algorithms. Unfortunately, the fitness function computation may be extremely time-consuming when dealing with antenna designs. Therefore, it is of paramount importance to speed up the convergency and to improve the performance of this kind of algorithm. The recent introduction and the increasing availability of quantum computing may be very effective in accelerating the design process, even though new approaches and algorithms are needed in order to exploit the specificity of these instruments. In this paper, a new version of a novel quantum crossover operator for binary Genetic Algorithm (bGA) has been introduced and compared with its previous version. They have been successfully tested on different mathematical benchmark functions and on a preliminary thinned array design.

**Keywords**—quantum computing operators, quantum crossover, binary genetic algorithms, electromagnetic optimization, thinned array

## I. INTRODUCTION

Evolutionary Algorithms (EAs) are extremely flexible and effective for solving binary optimization problems with multimodal and nonconvex cost functions. These are commonly present in many electromagnetic problems, explaining their popularity in the field, as it is shown in [1] and [2]. In order to manage effectively the design of complex radiating structures, with hundreds or even thousands of free design parameters, the convergency rate of EAs has to be improved [3], and several alternative solutions have been proposed and investigated i.e. in [4].

In fact, even though EAs have been achieving impressive features, there are still restrictions to their application due to the fact that the group of problems that can be solved efficiently in a Deterministic Turing Machine (DTM), our classical computers where we implement them, is limited. In computational complexity theory, the set P contains all decision problems that can be solved by a DTM [5,6]. In order to go beyond this limitation, in the last years, quantum computers have started to be first theoretically designed, and then physically introduced.

Even if their practical availability is still quite challenging, due to a set of technological limitations that are being

effectively addressed by different scientific communities, it is expected that they will be massively adopted in the next years, since quantum computers can solve a group of problems called BQP, that contain P [7]. In this sense, quantum computing is less limited than classical computing and this allows quantum computers to find better solutions to complex optimization problems faster, either rephrased in binary or continuous form. These advantages add to the fast development of real quantum computers [8], which boosted the development of novel optimization algorithms using quantum computing.

The properties of the quantum world are not by any means new to the optimization field i.e. they have been already investigated in [9] using the behavior of quantum particles in PSO, achieving a step increase in the algorithm performance in mathematical optimization tests (MOT). Nonetheless, the application of quantum computing is relatively new, being the selection algorithm presented in [10] one of the first works that uses real quantum circuits (QC) running in a quantum computer.

In any case, the introduction of quantum computing techniques in the optimization of EM problems has yet to be investigated in more detail, even if they already have been considered in [12]. This is because it is not straightforward to implement classical algorithms in quantum circuits, but on the contrary, it is important to develop a suitable alternative set of algorithms that can effectively exploit the specific properties of quantum computing.

The purpose of this paper is to introduce a new and improved version of an already presented Quantum Crossover Operator [13] and investigate its properties in the evolutionary optimization for electromagnetic design framework, considering in detail the performance of both versions in two mathematical test functions and an EM problem.

The whole paper is structured in seven sections. In the next sections, the binary Genetic Algorithm adopted as a common reference is presented, then the Quantum Crossover Operator's original version and new version are presented in III and IV. In section V all of them are compared in terms of performances on different mathematical benchmark test functions. Section VI presents the electromagnetic problem considered in detail, a thinned array design, and the optimized design is reported before a brief conclusion in Section VII.

## II. THE CONSIDERED GENETIC ALGORITHM

The bGA considered as a reference for this paper has been implemented employing as a base the procedures described in [3]. Its general flowchart is presented in Fig. 1. The introduced bGA has a population of 32 individuals, a two-point heuristic crossover, tournament selection, and a standard binary mutation. Additionally, in this paper, the terminal condition is given by a desired number of generations.

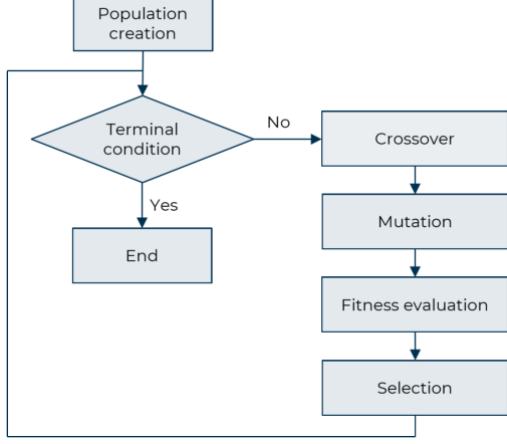


Fig. 1. The bGA flowchart.

## III. PRESENTATION OF THE QUANTUM CROSSOVER OPERATOR (BGA-QCO)

The bGA-QCO is the first iteration of the algorithm that is being introduced in this paper. It was originally presented in [13]. This algorithm starts from the basic bGA shown in section II and replaces the two-point heuristic crossover with a new operator that uses a quantum circuit to produce the offspring. The general flowchart of the crossover operator is presented in Fig. 2.

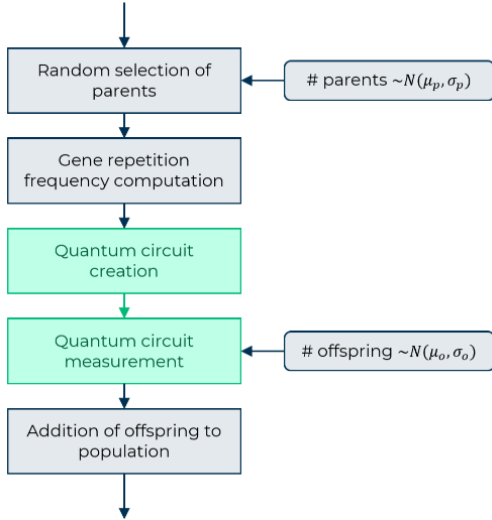


Fig. 2. QCO flowchart.

The way this works is by selecting randomly  $N$  parents from the population, and then the gene repetition frequency (GRF) is computed using the parents' genetic information. After this, the GRF is applied in the quantum circuit (QC) presented in Fig. 3. Finally,  $M$  measurements are done on the QC yielding  $M$  offspring that are added to the original population.

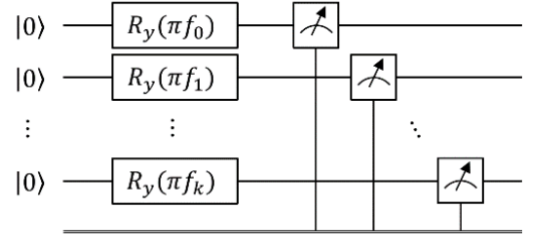


Fig. 3. Quantum circuit used to bGA-QCO.

The numbers  $N$  and  $M$  are normally distributed random numbers with mean  $\mu_p, \mu_o$  and standard deviation  $\sigma_p, \sigma_o$  respectively. The idea behind using random numbers is to add diversity to the parent pool size and to the offspring pool size. The GRF is computed using the times that a gene is present in the parents divided by  $N$ . An example of this process is presented in Fig. 4.

Parent 1	[0	1	1	...	0]
Parent 2	[1	1	0	...	1]
Parent 3	[1	1	0	...	0]
Parent 4	[1	1	0	...	1]

$$f = \left[ \frac{3}{4} \quad 1 \quad \frac{1}{4} \quad \dots \quad \frac{2}{4} \right]$$

Fig. 4. GRF computation example

## IV. INTRODUCING THE NEW VERSION OF THE BGA-QCO (BGA-GCO v2)

While working on the original version of the bGA-QCO it was noticed that it was not capable of handling thinned arrays with a size bigger than  $6 \times 6$ . This was attributed to a low genetic diversity caused by the forcefully rapid convergence of the original version. In order to overcome this problem, the new version presented here creates two subgroups of parents instead of one. Aiming to achieve a diverse group of offspring created by both subgroups of parents and therefore, boosting the overall genetic diversity of the population.

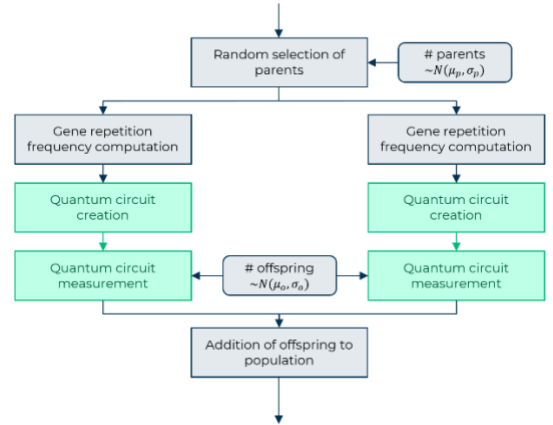


Fig. 5. QCO v2 flowchart.

The flowchart of this new version is presented in Fig. 5, the way the value  $N$  and  $M$  is computed is the same but  $\mu_p$  is set to a lower value than in the previous version in order to decrease the probability of having repeated parents. The  $\mu_o$  is kept the same because after selection the population is trimmed down to the original population size.

## V. ALGORITHMS COMPARISON ON MATHEMATICAL BENCHMARK FUNCTIONS

The three algorithms are implemented in Python using the libraries Numpy, Qiskit, and Matplotlib. The computer used for all the simulations is a laptop that has an Intel Core i7-10710U, an Nvidia GTX GeForce 1050Ti, 16 GB of RAM, and, 2 TB of SSD. For all the considered mathematical functions, the tests were run with the following parameters:

- Binary mutation rate = 0.01
- Generations = 100
- Dimensions = 6
- Precision of the binary number = 10
- Number of test = 100
- Parents mean value  $\mu_p = 20.8$  and 14.4 for v2
- Parents standard deviation  $\sigma_p = 5.6$
- Offspring mean value  $\mu_o = 8$
- Offspring standard deviation  $\sigma_o = 1.3$

The mathematical functions considered are Griewank, and Rastrigin, commonly used as benchmarks for testing the convergency of evolutionary algorithms. These two are selected due to their high complexity while offering different behavior in the transition between local minima and local maxima; Being smoother in Griewank and sharper in Rastrigin. For a further increase in complexity, the dimension is set to 6 to force the algorithm while keeping the number of qubits used to a reasonable value.

The parameters previously presented were obtained while testing the algorithm. It was found that a high number of parents gives less significance to the individuals with high fitness while a low number of parents can pass unwanted characteristics to the offspring. On the number of offspring, a high number decreases the genetic diversity of the population, while a low number decreases convergence speed. Therefore, they were set considering these observed behaviors using the standard deviation as a secondary tuning value.

For what concerns the convergency of the algorithms dealing with the 6-dimensional Griewank function, the obtained results are depicted in Fig.7 and summed up in Tab.1.

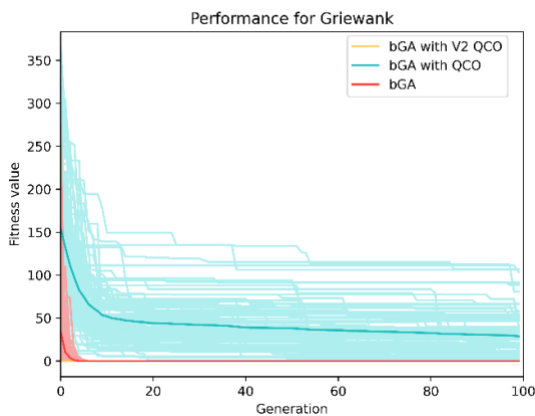


Fig. 7. Griewank 6-dimension convergency curves.

Algorithm	Best	Mean	Std. Dev
bGA	0.325	28.879	23.462
bGA-QCO	0	0	0
bGA-QCO v2	0	0	0

Table 1. Griewank 6-dimension convergency data

From this test, the overall performance of the two versions of bGA-QCO is far superior to that of the reference bGA obtaining an increased in speed to reach the optimum of multiple orders of magnitude. Between bGA-QCO and bGA-QCO v2 there is a slight improvement in the number of generations needed to arrive to the minimum moving from 5 or 6 to 2 or 3 in the new version.

Fig. 8 and Tab. 2 report respectively the results for the 6-dimension Rastrigin function, convergency curves and statistical data.

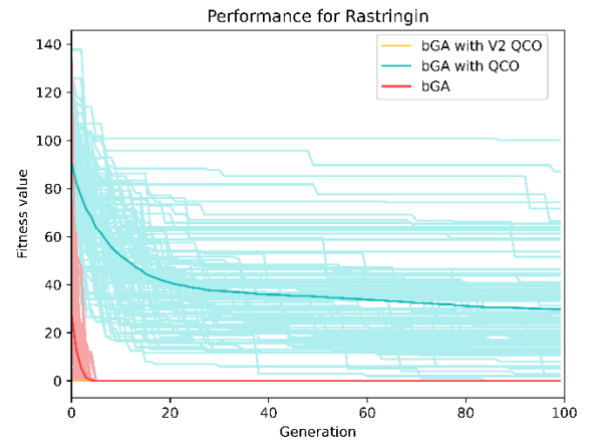


Fig. 8. Rastrigin 6-dimension convergency curves

Algorithm	Best	Mean	Std. Dev
bGA	0	29.716	18.939
bGA-QCO	0	0	0
bGA-QCO v2	0	0	0

Table 2. Rastrigin 6-dimension convergency data

Note that for this test function, the advantage of the quantum optimization techniques is more noticeable than for Griewank. The increase in performance of the new version is also noticeable in this case giving the overall edge to the new version over its predecessor for the mathematical test functions used. The fact that this optimization can reach the optimum consistently is significant but not unique since other optimization functions like a modified QPSO are able to reach the optimum in mathematical test functions. [14]

## VI. THE ELECTROMAGNETIC OPTIMIZATION PROBLEM: A THINNED ARRAY DESIGN

Binary optimization is very important in the antenna field where continuous problems may be rephrased to binary terms, or are intrinsically binary. Among these intrinsically binary problems are the thinned arrays [15], which are relevant in real-world applications since they can be used with different shapes and for different purposes.

Thinned arrays can achieve excellent performances compared with the corresponding full arrays by decreasing the number

of radiating elements and, thus the fabrication and transportation cost of the array, the complexity of the feeding network, and the overall weight of the system [15].

Antenna arrays are commonly introduced to have specific radiation performance by combining equal simple radiating elements. The simplest way to define an array of antennas is to take a regular grid and place a radiating element in each position of the grid. The array containing all the elements is called full array.

In this specific problem, the full array consists of a grid of  $8 \times 8$  equally spaced radiating elements. For the previous version of the bGA-QCO, the optimization was restricted to a  $6 \times 6$  array as it is shown in [13]. Given the array design, it is possible to calculate the radiation pattern using the standard array theory [16,17].

The complexity of a full array is high because it is necessary to design a feeding network for all the elements, taking into consideration that they should be fed with a correctly-phased signal.

In order to reduce the feeding network complexity it is possible to reduce the number of the radiating elements, i.e. to thin the array. By properly selecting the radiating elements, it is possible to tune the radiation pattern of the array and therefore, simplify the feeding network with an additional reduction of both weight and costs, which is important for multiple fields like aerospace applications.

This optimization problem aims to reduce the number of radiating elements and fit the radiation pattern to a mask given by a side lobe level of  $-20$  dB, with a main lobe of at most  $0.17$  rad.

The cost function used to compute the fitness of a solution is found using two sub-functions. The first one is the integral of the difference between the computed radiation pattern of the considered solution over all the solid angles,  $P = P(\theta, \varphi)$ , minus the desired mask,  $M = M(\theta, \varphi)$ . The second one is the number of elements in the array. Since both parameters had to be minimized, they were added together with a specific weight in order to give both of them a similar impact on the final fitness value. In addition, a constraint is applied in order to penalize the violation of the mask.

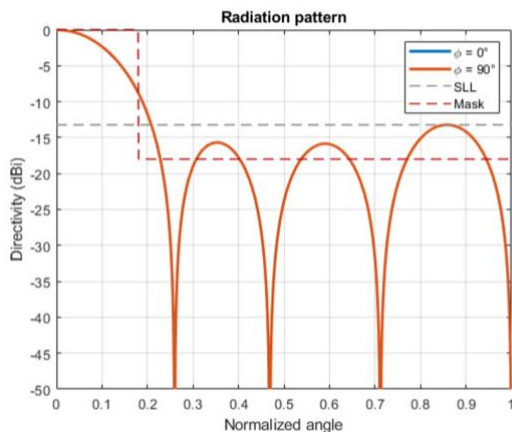


Fig. 9. Radiation pattern of the optimum thinned array obtained with bGA-QCO v2.

In Fig. 9, the radiation pattern for the obtained thinned array is presented, it is clear that it is not capable of achieving the mask due to its high constrain for a small array like this

one, but it is capable of getting a better result than the one achieved in previous versions [13] presented in Fig. 10.

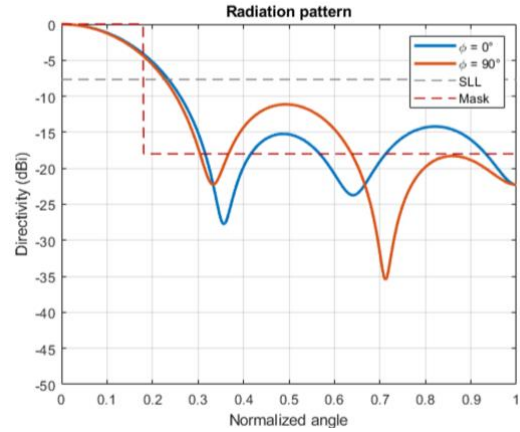


Fig. 10. Radiation pattern of the thinned array presented in [13].

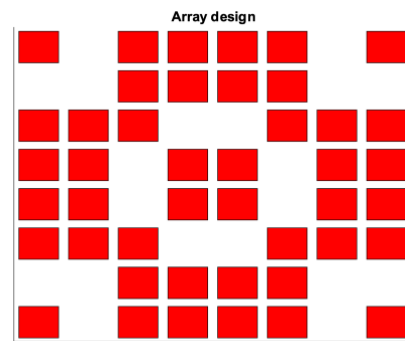


Fig. 11. Obtained optimum thinned array from bGA-QCO v2 optimization.

The thinned array obtained is presented in Fig. 11 and its convergence curve is presented in Fig. 12. From the results, it is clear that the optimization found that a symmetric solution was the best suited for the constraints given. Furthermore, what is assumed to be the minimum is achieved in at least 12% of the runs in less than 100 generations showing quick and reliable convergence for a computationally intensive optimization like this EM problem.

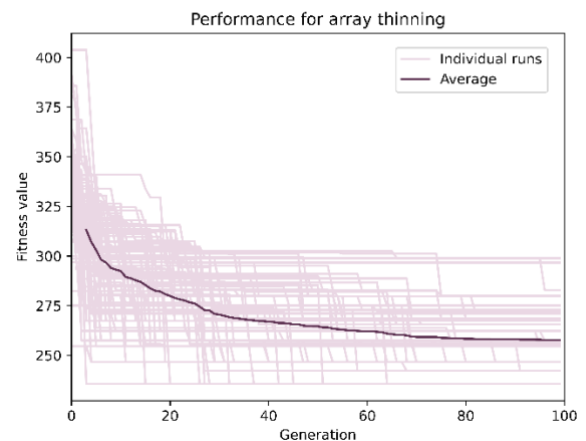


Fig. 12. Convergency curve of the EM problem.

The statistics of this optimization are presented in Tab. 3, from these it is also clear that most of the runs have the tendency to arrive at the minimum or at least a close value to the minimum in some additional generations.

Algorithm	Best	Mean	Std. Dev
bGA-QCO v2	235.624	257.657	14.701

Table 3. EM problem convergency data.

## VII. CONCLUSIONS

In this paper, an improved technique to introduce quantum computing in the evolutionary optimization of electromagnetic binary structures has been considered and investigated.

The new version of bGA-QCO has been compared to its predecessor and a conventional bGA. From this comparison, it became clear that it keeps its predecessor advantages while further improving its convergence speed.

Furthermore, it was tested with a problem that the first version was not capable handle, and it yielded outstanding results achieving a better fit to the mask while also eliminating more than 31% of the elements of the array improving its EM performance with respect to its predecessor while lowering its fabrication cost and complexity with respect to a full array.

The strategy taken to overcome the decrease in genetic diversity used in bGA-QCO v2 was successful letting the new version handle bigger problems with an increase in convergence speed.

The performance of the bGA-QCO v2 shows that it can be tested in other problems expecting good results and further proves the advantages that the introduction of quantum computing brings to the field of optimization especially for complex problems like the ones commonly encountered in electromagnetics.

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