Mixed Strategy to Cover A Convex WSN

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***Abstract*—In this paper, we have considered the coverage problem in wireless sensor network (WSN) on a convex subset of R^2. Sensors are dropped from the air randomly on some pre-fixed points, which is known as vertices, of Region of Interest (ROI). We use optimal partition of the ROI, which is actually partition in several regular hexagons. Since sensors are distributed randomly, a sensor may not be placed on the target vertex. For this reason, ROI will not be completely covered by a set of sensors. In practice, few more sensors are deployed on few (randomly chosen) vertices or used actuator (it can carry sensors to the proper vertex) to reduce the uncovered region or area. In one of our previous works, we have developed a strategy as follows: reduce the distance among two adjacent vertices and deployed one sensor on a vertex so that total number of sensors will be same as in existing old method (drop two sensors on some vertices and one sensor on the rest). We have compared the proportion of uncovered region using the commonly used old strategy with our previous one. We have simulated for several values of percentage of extra sensors and observed that our previous strategy is better for low standard deviation (s.d.), but not better for higher s.d. in both two and three-dimension. Inspiring from the above fact, in this paper, we combined above two strategies to find a general one, for deploying sensors in two-dimension. The excess sensors are divides in two parts. One part is used for decrease the side of the regular hexagon and other part is used for using one more sensor on some selected points. We simulate uncovered area and results indicate the optimal choice of these two parts, which change with the standard deviation of randomness.**

***Index Terms*—Coverage problem, random deployment, wireless sensor networks.**

I. Introduction

The main goal of a Wireless Sensor Network (WSN) is tracking an event in the domain of interest. WSNs are using in different area of civilization [18]–[20]. They are useful in image processing and for data storage also [1]. Clearly, sensors should cover the region (ROI) completely, without any holes. These holes are called sensing hole. Sensor can detect objects or event in a circular disc (called, sensing disc) of a particular radius (called, sensing radius). A point of ROI is not be covered by sensors when that point does not lie on the sensing disc of any of these sensors. Hence, we have to place the sensors in a way that there must not be any sensing hole in the ROI. In some networks robot(s) is used to cover the ROI [13], but here we assume there is no robots.

However, we cannot confirm that, the sensors will be placed at the target point (known as vertex), since the sensors randomly deployed from the air [5]. There are other causes of wrong placement also [17]. Here we assume that sensors are deployed in a convex bounded set of R^2. We want to develop a strategy for dropping of sensors on ROI to minimize the area of sensing holes.

There are two different methods of placement of sensors: (i) deterministic way and (ii) random deploying on target points [2]. Using method (i), we can cover ROI fully by sufficient number sensor. Using method (ii), it is not possible to cover the full region unless we use actuator. Here we use the second method.

After the deployment of sensors, sometime we use actuator to cover the uncovered region of ROI. This type of network is known as wireless sensor and actuator network [4], [16]. In other networks, there are some sensors which can move, and relocate at nearby point without any help [1], [8]. Since movable sensor requires huge amount of energy, actuator assisted sensor placement is more useful.

There are several methods to deploy sensors to cover a convex and bounded set of R^2. This type of problem in sensor network is usually called coverage problem. Several variations of this problem can be found in [3], [14], [15]. A survey on different strategies of deployment is in [7]. Analysis of the expected and maximum distance covered by actuator to achieve the coverage is discussed in [6], [9]. When ROI is square grid, the coverage problem is a graph theoretic problem [11]. Nandi and Li develop a new algorithm using one actuator to cover the uncovered region [17]. They describe two randomized algorithms in case of grid structure. They simulate and analyzed distance travelled by actuators.

A. Motivation

The most important question in WSN is ‘Is ROI fully covered or not? and, if the ROI is not fully covered; then how we cover the region using extra sensors’. Many works have been done on this, but only few works have been done on the problems: (i) 'how the proportion of the uncovered region is related with respect to the number of sensors?' and (ii) 'how the uncovered region depends on the strategy of placements of the sensors?'.

Since the sensors are deployed randomly, it is not guaranteed that the region is completely covered even if we use sufficient number of sensors, unless one relocates of those sensors, using movable sensors or using actuator. Here we consider no movable sensor or actuator are used. Our aim is to find a strategy for deployment of sensors for decrease the uncovered proportion.

Note that, a point of the region is sufficient to cover by exactly one sensor. Hence, if a point of the region is covered by more than one sensor; it is some sense wastage of sensors. Since covered region by a sensor is circular, one cannot avoid wastage. Our goal is to reduce the wastage portion. Usually, we place sensors in a deterministic way on the pre-fixed points of ROI, so that the wastage is minimum. After random deployment, there must be some uncovered area in ROI. So, we need extra sensors.

So, the problem is, how do we deploy sensors so that wastage is minimum or at least reduced compare to the existing methods. Nandi et.al. find a strategy to solve that problem in R^2 [8] and in R^3 [10], [12]. To solve the problem in two-dimensions, we divide the whole region into required number of regular hexagons, since, in the case of deterministic placement, coverage is best when we divide region into congruent regular hexagons. But in case of random deployment, the hexagonal division may not be best among all types of partition. In the above two papers, they consider the hexagonal and cube centered partition in two and three dimensions respectively.

In above papers, authors assumed that sensor may be placed at any arbitrary point of the region of interest. They also assumed, the distance D_i between two points; (1) where the i-th sensor actually placed; and (2) the target point where the i-th sensor pre-fixed to place; is random. Now when we placed some extra sensors at some random points of the region, the uncovered area will be decrease. On the other hand, if we deploy exactly one sensor at each of the target point, and decrease the distance between two neighbouring target points, then also the uncovered area will be decrease. In either situation we keep the sensing radius and number of sensors same. Comparisons between the above two methods are as follows:

(1) In first strategy (call it S_1), we target to deploy two sensors on some randomly chosen vertices and one sensor on remaining vertices. In second strategy (call it S_2), we deploy exactly one sensor on all vertices.

(2) Let in S_1, there are m regular hexagons and l extra sensors, i.e., total $m+l$ sensors is placed in S.. If length of side of the hexagon is b in S_1, then in S_2 length of side of the hexagon is c where, $(m+l)c^2 = mb^2$. Hence the total covered area is equal in either case. The distance between two adjacent target points is less for S_2 than that of S_1.

In above papers, authors consider the region is a convex subset of R^2 or R^3. The distance (D_i) is a random variable. We have assumed that the probability distribution of (D_i) is either normal or uniform. The authors simulated the uncovered area for above distributions and for two strategies S_1 and S_2. And it found that S_1 is better for distribution which have higher standard deviation and S_2 is better for distribution which has smaller standard deviation.

Since the lower and higher standard deviation (s.d.) distribution behaves oppositely for two strategies, we hope that in case of moderate (i.e., between higher and lower) s.d. mixing of the above two strategies may be better. In this new mixed strategy, the idea is; divide all extra sensors in two parts: (1) One set of sensors is used for reducing the distance between the two adjacent vertices (i.e., using our previous method) and (2) the set of remain sensors is used for dropping two sensors on randomly selected vertices (i.e., using usual method). The percentage of division into two parts depends on the s.d., and hope that there is a optimal division of two parts depending on the corresponding s.d. Here we divide the ROI into regular hexagon. Simulation results show our hypotheses is correct. Results also suggest the optimal division of two parts. We consider uniform, normal distributions and a nonparametric distribution.

II. Assumptions and Definitions:

In this paper we consider a bounded convex set in two dimensions as region of interest (ROI). Assume the region of interest is partitioned into congruent regular hexagons. Length of the side of these hexagons is a. To cover a hexagon by one sensor one should take $a \leq r$, where r is sensing radius. When $r = a$ each hexagon will be covered by a sensor (considering the coverage area of sensor as inscribed regular hexagon of the sensing disc) when the sensor is on the center of that particular hexagon. Since a sensor is too small, so one can assume sensor as a point.

More mathematically, for an indexed set T, let the set of unit discs $\{C_T \subset R^2 : t \in T\}$, which can cover a set in R^2. This set to be considered as ROI. Let collection of two-dimensional random vectors be $\{Y_t : t \in T\}$. Let D_t be the distance between Y_t and the center of C_t, for $t \in T$. Assuming that D_t's are i.i.d., we consider three probability distributions for D_t here, one is nonparametric and other two are uniform and normal.

Let there are n regular hexagons (before reducing the side length) and $n+l$ (with l extra sensors) sensors. Let after reducing the side length there are $n+k$ (where $k <$

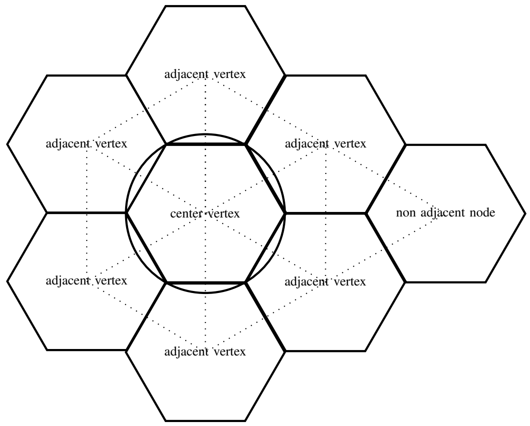


Fig. 1. Vertices of ROI partitioned into regular hexagons

l) regular hexagons. $2(l-k)$ sensors are used to deploy 2 sensors on $l-k$ random vertices and $n-l+2k$ sensors are used to deploy 1 sensor on remaining $n-l+2k$ vertices. Initially length of the sides is a. After reducing the length of the side is changed to b where $(n+k)b^2 = na^2$. Hence the area covered is be equal for both strategies.

Here are some important definitions:

- **Node:** The point in the ROI where a sensor is placed after random deployment is known as node. We also use the word 'node' to mean the respective sensor.
- **Vertex:** The particular point in the ROI where a sensor is to target to deploy is known as vertex.
- $N(V)$ is the corresponding node of the vertex V. Hence when a sensor targeted to place on V it was placed on $N(V)$. Observe, when there is no randomness, then a node and its corresponding vertex is same, i.e., $N(V) = V$.
- **Sensing disc and sensing radius:** The closed disc with center N and radius r is known as sensing disc. The disc corresponds to the node N will be denoted by S_N. So, S_N will be covered the sensor placed at the node N. The radius r is known as sensing radius. Sensing radius is same for all nodes.
- **Adjacent vertex:** It is a vertex which is at distance less than $2r$ from the particular vertex. Clearly, the sensing disc S_N of a node N has no intersection with the sensing disc of a node M which is not an adjacent node of N.
- The distance between any two points A and B in ROI will be denoted by $d(A, B)$.
- Denote set of all nodes as $\mathcal{N}$. A point $A \in \mathbb{R}^2$ is said to be covered by a node N if $d(A, N) \leq r$ and the point A is said to be covered by the set $\mathcal{N}$ of nodes, if A is covered by at least one node of $\mathcal{N}$. A point is said to be uncovered by N if it not covered by N and the point is said to be uncovered by $\mathcal{N}$ if it is uncovered by all nodes in $\mathcal{N}$.
- **Sensing hole:** A connected subset of ROI is a known to be sensing hole, whose every points are uncovered by $\mathcal{N}$.
- **Adjacent sensing hole** of a node is that sensing hole whose boundary have an intersection with the boundary of the corresponding sensor disc.
- ROI is **covered by a set of nodes** $\mathcal{N}$ if each point in ROI is covered by $\mathcal{N}$.
- Area of a bounded subset S of $\mathbb{R}^2$ is denoted by **Area** (S).

We shall now define the most important term called **wastage**. Let S be any bounded set in $\mathbb{R}^2$, which is covered by a finite set of sensors or nodes $\mathcal{N}$. The wastage in S for $\mathcal{N}$ is define as follows:

$$W_{\mathcal{N}}(S) = \frac{\sum_{M \in \mathcal{N}} \text{Area}(S \cap S_M) - \text{Area}(S)}{\sum_{M \in \mathcal{N}} \text{Area}(S \cap S_M)} \cdots (1)$$

If $\mathcal{N}$ is a set so that $|S_{N_1} \cap S_{N_2} \cap S_{N_3}| \leq 1$ for distinct $N_1, N_2, N_3 \in \mathcal{N}$, then

$$W_{\mathcal{N}}(S) = \frac{\sum_{N_1 \neq N_2 \in \mathcal{N}} \text{Area}(S \cap S_{N_1} \cap S_{N_2})}{\sum_{M \in \mathcal{N}} \text{Area}(S \cap S_M)} \cdots (2)$$

Intuitively, the denominator of the above expression is the total area, which is common with S, and the numerator is the wastage in volume. Here 'wastage' represent the ratio of wastage area and the total area.

III. Simulation Result of mixed Strategy

In this section, we shall describe the simulation process and the data and results what we got from that. The system parameters which are involve in simulation are:

- **radius of sensing disc (r):** Since r has no effect in the simulation study, we take $r = 1$.
- **Number of vertices:** we have partitioned the ROI as congruent regular 10000 hexagons and deploy one sensor at the center (vertices) of all the regular hexagons. Total number of nodes is also 10000 unless more than one sensor falls in the same point (which has 0 probability).
  Total area considered in simulation is $10000 \times \frac{3\sqrt{3}}{2}$ unit. Two adjacent centre or vertices has distance $\sqrt{3}$ units. 100 sensors will be placed on a row of the grid.
- **Standard deviation σ of the distribution of D_i:** We consider the distance D_i's are i.i.d. either uniform or normal or a non-parametric distribution. The uniform distribution whose p.d.f. is $f(x) = \frac{2x}{\sigma^2} I_{(0,\sigma)}$ denoted by $U(\sigma)$ and $N(0, \sigma^2)$ be the normal distribution with expectation 0 and standard

deviation σ. We take several different values for σ in our simulation.

- **Number of extra sensors** p_1, p_2 **and** p**:** Let $100p_1$ extra sensors are used to reduce the distance between two adjacent centre and $100p_2$ extra sensors are used to deploy 2 sensors on $100p_2$ random vertices. Hence $100p$ (where $p = p_1 + p_2$) extra sensors are used and total number of sensors is $10000 \times \left(1 + \frac{p_1+p_2}{100}\right)$ where $p, p_1, p_2 \in [0, 100]$.

A. *Simulation method in flow chart:*

In this section, we describe our proposed strategy. Our new mixed strategy is as follows:

- **Step 1:** Partition ROI into $10000\left(1 + \frac{p_1}{100}\right)$ congruent regular hexagon of side $\sqrt{\frac{100}{100+p_1}}$. Consider $10000\left(1 + \frac{p_1}{100}\right)$ centers of these hexagons as vertices and deploy one sensor exactly for every vertices. Area of the whole region is $10000\left(1 + \frac{p_1}{100}\right) \times \frac{3\sqrt{3}}{2}\left(\sqrt{\frac{100}{100+p_1}}\right)^2 = 10000 \times \frac{3\sqrt{3}}{2}$.
- **Step 2:** Select $100p_2$ vertices randomly and uniformly from the $10000\left(1 + \frac{p_1}{100}\right)$ and deploy two sensors onto each of selected vertices and deploy 1 sensor onto other $(10000+100p_1-100p_2)$ vertices.
- **Step 3:** Simulated the ratio of uncovered region or area of ROI with different values of p_1, p_2 and t for uniform, normal and for a non-parametric distribution, where p_1, p_2, t lies between 0 and 1.
- **Step 4:** Repeat the process step 1 to step 3, for fixed value of the parameters p_1, p_2 and σ for 10000 many times and found the average of the ratios.
- **Step 5:** Repeated the whole process (step 1 to step 4) to find the average of ratios for different values of the parameters p_1, p_2, σ.

B. *Simulation data and observations:*

We have the following tables I to IV showing the average proportion of uncovered area with different values of p_1, p_2 for $U(0.5), U(1.0), N(0, 0.1), N(0, 0.25)$ respectively. First row represents p_1 and first column represents p_2.

From the table one can find the best choice of the p_1 and p_2 for a particular value of $p = p_1 + p_2$ for four different distributions. E.g., consider the distribution $U(1.0)$. For $p = 5$ when $p_1 = 0$ and $p_2 = 5$ the proportion of covered area is 0.92904, whereas when $p_1 = 5$ and $p_2 = 0$ the proportion of covered area is 0.92918. Hence the choice of $p_1 = 5$ and $p_2 = 0$ is better than the previous one.

Next consider distribution $N(0, 0.1)$. For $p = 40$ when $p_1 = 10$ and $p_2 = 30$ the proportion of covered area is 0.99003, which is the best choice of p_1 and p_2. Clearly if the length of intervals of p_1 and p_2 are shortened we shall get precise optimal choice of p_1, p_2.

We have drawn the 'proportion of covered area (δ)' vs. '$p/100$' graphs for five distributions and three different values of p_1 and p_2 (see Figure 2a to 2e). The red, blue and black curves show the values of proportion of covered area for different values of p and for $(p_1, p_2) = (p, 0), (p_1, p_2) = (p/2, p/2)$ and $(p_1, p_2) = (0, p)$ respectively. From the curves we can find the optimal choice of p_1 and p_2. We can draw more curves, for different choices of p_1 and p_2 similarly, to get better choice of p_1 and p_2.

Comparison between two existing strategies and proposed strategy

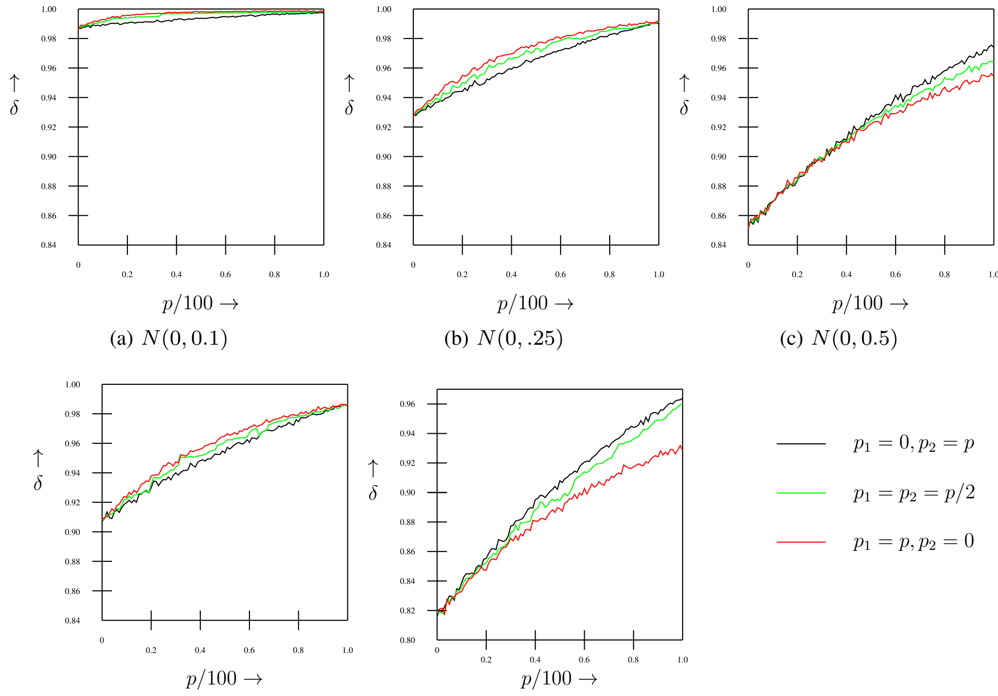
Proposed strategy is a generalization of two existing strategy (our previous strategy and commonly used strategy). This strategy is mixture (like convex combination) of two existing strategy. So, if for some choice of parameters one of the old strategies is better, then our new mixed strategy shows that fact. But in many situations our new strategy shows how to mixed two previous strategies to find the optimal strategy.

IV. Conclusion

In this article, we have introduced a new idea in the coverage problem on a bounded convex set in R^2. We consider that each sensor is deployed randomly, hence there is very low chance to place the sensor on the target position. Note that it can be placed at an arbitrary position in the ROI. The distance between the two points (the target point and the point where the sensor is dropped) is a random variable and follow i.i.d. uniform or bivariate normal or a nonparametric distribution. For those distributions we simulate the uncovered region or area using a C-code.

We develop a new strategy for using sensors which is a mixture of two previous strategies to reduce the uncovered region or area than that of the previous methods. We have compared this mixed strategy with previous strategies. We consider hexagonal partition of ROI. We divide the extra sensors in two groups. One is used for reducing the length of hexagon and the other is used to deploy two sensors on some randomly selected points. Simulation data and results show the mixed strategy works better. Simulation also suggests the optimal division of two parts.

In future work we shall consider the deployment of sensors for three dimensions. One may divide ROI into square. One can consider other parametric distributions like multivariate normal or exponential probability distribution for D_i. In this article we consider a mixed strategy to deploy sensors, which is better among all existing strategies for the above distributions. One can find other strategies also, which may be better for different distributions.



(a) $N(0, 0.1)$ (b) $N(0, .25)$ (c) $N(0, 0.5)$ (d) $U(0.5)$ (e) $U(1.0)$

Fig. 2. Graph (from simulation data) of proportion of coverage area in $\mathbb{R}^2$

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TABLE I

PROPORTION OF THE COVERAGE AREA FOR $U(1.0)$

$p_1 \rightarrow$	0	5	10	15	20	25	30	35	40	45	50
0	0.92341	0.92918	0.93350	0.93680	0.94500	0.94876	0.95321	0.95676	0.95753	0.95932	0.96247
5	0.92904	0.93412	0.93762	0.94519	0.94901	0.95410	0.95782	0.95799	0.96071	0.96350	0.96765
10	0.93380	0.93942	0.94597	0.94998	0.95512	0.95802	0.95843	0.96226	0.96454	0.96854	0.97139
15	0.94231	0.94625	0.95016	0.95601	0.95897	0.95957	0.96352	0.96589	0.96932	0.97199	0.97731
20	0.94601	0.95321	0.95754	0.95964	0.96187	0.96432	0.96672	0.97031	0.97267	0.97719	0.98001
25	0.95676	0.95807	0.96002	0.96352	0.96517	0.96749	0.97110	0.97318	0.97801	0.98062	0.98236
30	0.95997	0.96187	0.96529	0.96602	0.96831	0.97192	0.97400	0.97893	0.98135	0.98311	0.98471
35	0.96302	0.96754	0.96725	0.96932	0.97275	0.97532	0.97971	0.98211	0.98389	0.98542	0.98666
40	0.96896	0.96843	0.97076	0.97327	0.97687	0.98056	0.98301	0.98467	0.98608	0.98732	0.98831
45	0.97003	0.97197	0.97401	0.97802	0.98148	0.98399	0.98542	0.98692	0.98809	0.98902	0.99001
50	0.97300	0.97521	0.97901	0.98231	0.98489	0.98611	0.98755	0.98897	0.98999	0.99053	0.99102

TABLE II

PROPORTION OF THE COVERAGE AREA FOR $N(0, 0.1)$

$p_1 \rightarrow$	0	5	10	15	20	25	30	35	40	45	50
0	0.96569	0.96881	0.97079	0.97591	0.97649	0.97870	0.97976	0.98231	0.98421	0.98520	0.98694
5	0.96849	0.97021	0.97542	0.97772	0.97983	0.98001	0.98341	0.98534	0.98597	0.98705	0.98721
10	0.96951	0.97387	0.97875	0.98099	0.98081	0.98403	0.98607	0.98653	0.98749	0.98897	0.98911
15	0.97230	0.97997	0.98189	0.98205	0.98571	0.98701	0.98712	0.98764	0.98812	0.99001	0.99119
20	0.97968	0.98264	0.98310	0.98610	0.98826	0.99839	0.99841	0.98901	0.99100	0.99197	0.99208
25	0.98230	0.98421	0.98735	0.98921	0.98953	0.98960	0.98998	0.99241	0.99243	0.99241	0.99294
30	0.98543	0.98812	0.99033	0.99034	0.99035	0.99058	0.99098	0.99241	0.99278	0.99307	0.99452
35	0.98901	0.99021	0.99035	0.99036	0.99081	0.99149	0.99258	0.99301	0.99378	0.99498	0.99537
40	0.98997	0.99025	0.99041	0.99173	0.99261	0.99309	0.99372	0.99450	0.99501	0.99568	0.99619
45	0.99054	0.99060	0.99281	0.99324	0.99380	0.99421	0.99471	0.99513	0.99581	0.99674	0.99765
50	0.99132	0.99219	0.99310	0.99388	0.99412	0.99490	0.99541	0.99601	0.99721	0.99804	0.99900

TABLE III

PROPORTION OF THE COVERAGE AREA FOR $N(0, 0.5)$

$p_1 \rightarrow$	0	5	10	15	20	25	30	35	40	45	50
0	0.92843	0.93503	0.93502	0.93948	0.94021	0.94377	0.95132	0.95343	0.95676	0.95900	0.96105
5	0.93344	0.93473	0.94051	0.94579	0.95588	0.95639	0.95644	0.95814	0.96071	0.96176	0.96372
10	0.93647	0.94152	0.94758	0.94859	0.95364	0.95464	0.95688	0.96072	0.96172	0.96378	0.96574
15	0.94454	0.94660	0.94761	0.95264	0.95464	0.95868	0.96473	0.96734	0.96874	0.96875	0.96975
20	0.95061	0.95363	0.95655	0.95665	0.96096	0.96473	0.96574	0.96753	0.96876	0.96973	0.97074
25	0.95645	0.95866	0.95965	0.96050	0.96774	0.96875	0.96914	0.97069	0.97373	0.97783	0.97951
30	0.96071	0.96162	0.96715	0.96875	0.96954	0.97169	0.97376	0.97282	0.97590	0.97698	0.98002
35	0.96067	0.96712	0.96762	0.96877	0.96877	0.96978	0.97090	0.97298	0.97580	0.97814	0.98019
40	0.96705	0.96753	0.968782	0.96864	0.97093	0.97398	0.97580	0.97816	0.97822	0.97982	0.98032
45	0.97025	0.97289	0.97197	0.97802	0.97980	0.98016	0.98025	0.98130	0.98235	0.98341	0.98452
50	0.97083	0.97296	0.97384	0.97813	0.97825	0.98032	0.98138	0.98434	0.98309	0.98460	0.98563

TABLE IV

NON-PARAMETRIC DISTRIBUTION WITH S.D. 0.5

$p_1 \rightarrow$	0	5	10	15	20	25	30	35	40	45	50
0	0.92432	0.93032	0.93102	0.93482	0.93640	0.94077	0.94513	0.95043	0.95167	0.95390	0.95610
5	0.93044	0.93307	0.93405	0.93998	0.94588	0.95039	0.95432	0.95541	0.95607	0.96076	0.96221
10	0.93471	0.93941	0.94275	0.94591	0.94642	0.95046	0.95368	0.95607	0.95861	0.96037	0.96257
15	0.94254	0.94366	0.94612	0.95064	0.95146	0.95681	0.96147	0.96347	0.96428	0.96532	0.96750
20	0.94615	0.948536	0.95165	0.95266	0.95609	0.95739	0.96157	0.96331	0.96587	0.96737	0.96977
25	0.95452	0.95661	0.95796	0.95960	0.96277	0.96487	0.96791	0.96870	0.97137	0.97478	0.97695
30	0.95607	0.95861	0.96071	0.96287	0.96495	0.96716	0.97037	0.97182	0.97359	0.97569	0.97802
35	0.95706	0.95967	0.96176	0.96387	0.96587	0.96830	0.97000	0.97208	0.97458	0.97681	0.97893
40	0.96052	0.96343	0.96587	0.96648	0.96709	0.96818	0.97023	0.97163	0.97382	0.97598	0.97803
45	0.96702	0.96899	0.97097	0.97380	0.97598	0.97801	0.97980	0.98092	0.98123	0.98234	0.98401
50	0.96783	0.96961	0.97184	0.97339	0.97521	0.97803	0.98038	0.98240	0.98300	0.98346	0.98531