

# On the Fault-Tolerant Capability of a Class of Constructions of Optical Priority Queues

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**Abstract**—Constructions of optical buffers for conflict resolution among packets competing for the same resources is one of the most important and challenging issue in all-optical packet-switched networks. Priority queues is one of the most general and versatile buffering schemes, and includes the most commonly used first-in first-out (FIFO) queues and last-in first-out (LIFO) queues as special cases. The best constructions of optical priority queues currently available in the literature was given by Cheng *et al.* by using a feedback system consisting of an optical  $(\sum_{i=1}^k m_i n_i + 2) \times (\sum_{i=1}^k m_i n_i + 2)$  (bufferless) crossbar switch and  $k$  groups of optical FIFO multiplexers with delay one (FM1's), where the  $i^{\text{th}}$  group has  $m_i$  parallel optical  $n_i$ -to-1 FM1's ( $n_i$  FM1's) with the same buffer size  $B_i$  ( $B_i \geq 1$ ) for  $i = 1, 2, \dots, k$  (see Figure 2 in Section I). In this paper, we consider an important practical issue in the constructions of optical priority queues: the *fault-tolerant capability*. We consider the scenario that each group of FM1's has the same number of FM1's, say,  $m_i = m$  for  $1 \leq i \leq k$ , and show that the optimal construction obtained by Cheng *et al.* possesses fault-tolerant capability so that the feedback system can still be operated as an optical priority queue but with a *smaller* buffer size after up to  $f$  FM1's fail to function properly, where the fault-tolerant capability  $f$  can be expressed in terms of the number  $m$  of FM1's in each group and the numbers  $n_1, n_2, \dots, n_k$  of arrival links of the FM1's in the  $k$  groups as  $f = \min_{1 \leq i \leq k} \lfloor (m-1)/(n_i-1) \rfloor$ . Such a result can be used in the design of the parameters  $m$  and  $n_1, n_2, \dots, n_k$  to provide quality of service (QoS) that guarantees a certain level of fault-tolerant capability.

**Index Terms**—FIFO multiplexers, optical buffers, optical queues, optical switches, priority queues.

## I. INTRODUCTION

One of the most critically sought-after technologies in all-optical packet-switched networks is the design and implementation of optical buffers to overcome the currently existing optical-electrical-optical (O-E-O) hurdle due to the lack of *optical* random-access memory. A popular and feasible approach for the implementation of optical buffers is to use optical fiber delay lines as the storage media to store optical packets and use optical (bufferless) crossbar switches to route optical packets through the optical fiber delay lines [1]. By careful designs of the routing policy performed by the optical crossbar switches and careful choices of the delays of the optical fiber delay lines, one can route optical packets to the

right places at the right times and achieve *exact* emulations of the desired optical buffers.

In the last three decades, there have been extensive studies on Switched-Delay-Lines (SDL) constructions of optical buffers. These works include the early feasibility studies in [2]–[5], output-buffered switches in [6]–[11], first-in first-out (FIFO) multiplexers in [6] and [11]–[21], FIFO queues in [21]–[26], last-in first-out (LIFO) queues in [23], [24], and [27], priority queues in [28]–[39], time slot interchanges in [21] and [40], linear compressors/decompressors, non-overtaking delay lines, and flexible delay lines in [21] and [41]–[46], and FIFO/LIFO/absolute contractors in [47]. Moreover, results on the fundamental complexity of SDL constructions of optical queues can be found in [48] and performance analysis for optical queues has been addressed in [49] and [50]. For review articles on SDL constructions of optical buffers as well as related implementation and feasibility issues, we refer to [51]–[56] and the references therein.

In this paper, we consider SDL constructions of optical priority queues. Priority queues is one of the most general and versatile buffering schemes, and includes the most commonly used FIFO queues and LIFO queues as special cases, where the arrival time of a packet is used for the assignment of its priority. In a priority queue, each packet is associated with a *priority* upon its arrival, the packet with the *highest* priority is sent out from the queue whenever there is a departure request and there are packets in the queue, and the packet with the *lowest* priority is dropped from the queue whenever there is a buffer overflow.

Sarwate and Anantharam of UC Berkeley [28] are the first to propose constructions of optical priority queues by using a feedback system consisting of an optical  $(M+2) \times (M+2)$  (bufferless) crossbar switch and  $M$  optical fiber delay lines with properly chosen delays  $d_1, d_2, \dots, d_M$  (see Figure 1). The buffer size achieved in [28] is  $O(M^2)$ . By using better designs of the routing policy performed by the optical crossbar switch and better choices of the delays of the optical fiber delay lines in Figure 1, several improvements on the  $O(M^2)$  buffer size obtained in [28] have been made in [29]–[39]. The best result currently available was obtained by Cheng *et al.* in [37, Theorem 10] and [39, Theorem 12], and the buffer size achieved in [37] and [39] is  $2^{O(\sqrt{\alpha M})}$ , where  $\alpha$  is a constant

This work was supported in part by the Ministry of Science and Technology, Taiwan, under Contract MOST-111-2221-E007-041-MY2.

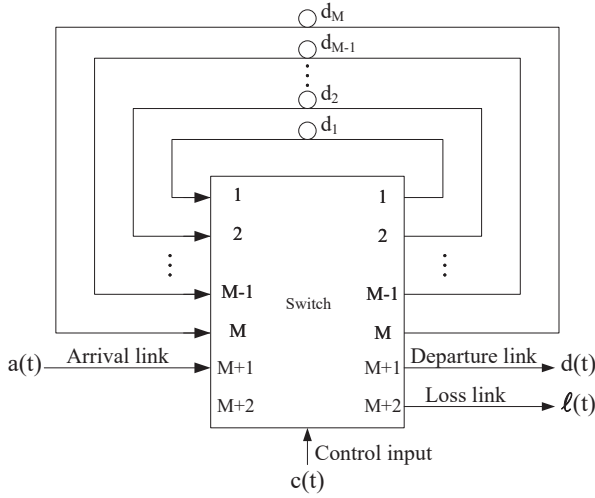


Fig. 1. A construction of an optical priority queue by using a feedback system consisting of an optical  $(M+2) \times (M+2)$  (bufferless) crossbar switch and  $M$  optical fiber delay lines with delays  $d_1, d_2, \dots, d_M$ .

that depends on the parameters used in their constructions. The buffer size  $2^{O(\sqrt{\alpha M})}$  is “exponential” in  $\sqrt{M}$  and significantly outperforms all previous results in [28]–[36] that are only “polynomial” in  $M$ .

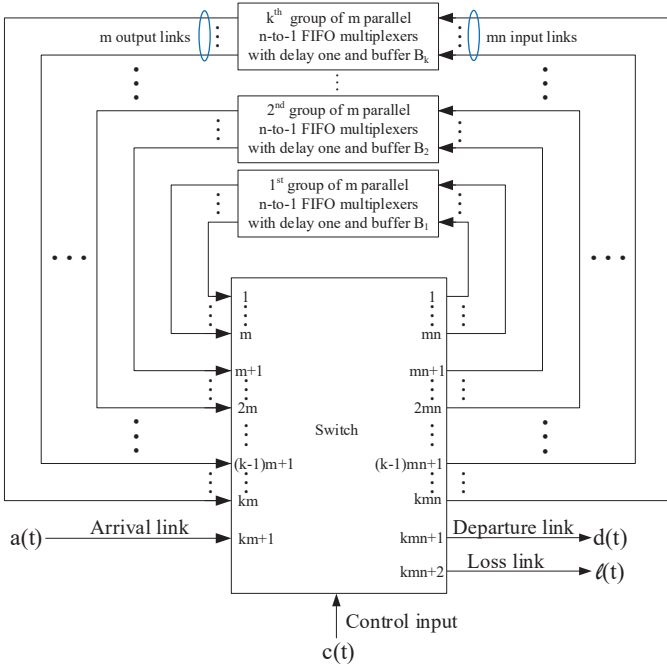


Fig. 2. A construction of an optical priority queue by using a feedback system consisting of an optical  $(\sum_{i=1}^k m_i n_i + 2) \times (\sum_{i=1}^k m_i n_i + 2)$  (bufferless) crossbar switch and  $k$  groups of optical FIFO multiplexers with delay one (FM1’s), where the  $i^{\text{th}}$  group has  $m_i$  parallel optical  $n_i$ -to-1 FM1’s ( $n_i$ FM1’s) with the same buffer size  $B_i$  ( $B_i \geq 1$ ) for  $i = 1, 2, \dots, k$ . For brevity, in this figure we denote  $M'_i = \sum_{j=1}^i m_j$  and  $M''_i = \sum_{j=1}^i m_j n_j$  for  $i = 1, 2, \dots, k$ .

The constructions in [39] use a feedback system consisting of an optical  $(\sum_{i=1}^k m_i n_i + 2) \times (\sum_{i=1}^k m_i n_i + 2)$  (bufferless)

crossbar switch and  $k$  groups of optical FIFO multiplexers with delay one (FM1’s), where the  $i^{\text{th}}$  group has  $m_i$  parallel optical  $n_i$ -to-1 FM1’s ( $n_i$ FM1’s) with the same buffer size  $B_i$  ( $B_i \geq 1$ ) for  $i = 1, 2, \dots, k$  (see Figure 2).

In this paper, we address an important practical issue in the constructions of optical priority queues: the *fault-tolerant capability*. We consider the scenario that each group of FM1’s has the same number of FM1’s, say,  $m_i = m$  for  $1 \leq i \leq k$ , and use the general constructions of optical priority queues obtained in [39] to show that the optimal construction in [39] possesses fault-tolerant capability so that the feedback system can still be operated as an optical priority queue but with a *smaller* buffer size after up to  $f$  FM1’s fail to function properly, where the fault-tolerant capability  $f$  can be expressed in terms of the number  $m$  of FM1’s in each group and the numbers  $n_1, n_2, \dots, n_k$  of arrival links of the FM1’s in the  $k$  groups as  $f = \min_{1 \leq i \leq k} \lfloor (m-1)/(n_i-1) \rfloor$  (see Theorem 3 in Section III). Such a result can be used in the design of the parameters  $m$  and  $n_1, n_2, \dots, n_k$  to provide quality of service (QoS) that guarantees a certain level of fault-tolerant capability.

The rest of the paper is organized as follows. In Section II, we give a formal definition of a priority queue and briefly review the constructions of optical priority queues in [39]. In Section III, we show that the optimal constructions in [39] possess fault-tolerant capability, and express the fault-tolerant capability in terms of the parameters used in the constructions. Finally, we conclude this paper in Section IV.

## II. REVIEW OF THE CONSTRUCTIONS OF OPTICAL PRIORITY QUEUES IN [37]

In this section, we first give a formal definition of a priority queue, and then review the constructions of optical priority queues in [39].

### A. Priority Queues

As in most works on SDL constructions of optical queues, in this paper we consider a discrete-time setting and make the following assumptions: (i) Time is *slotted* and *synchronized*. (ii) Packets are of the *same* size so that a packet can be transmitted through a link within a time slot. (iii) An optical  $M \times M$  (bufferless) crossbar switch is a network element with  $M$  input links and  $M$  output links that can realize all of the  $M!$  *permutations* between its inputs and its outputs. (iv) A fiber delay line with delay  $d$  is a network element with one input link and one output link that requires  $d$  time slots for a packet to traverse through. (v) There is at most one packet from any link of any network element at any time slot. (vi) Every network element is initially empty at time slot  $t = 0$ .

We note that *variable-size* packets can be taken care of with ease by implementing packet segmentation at the sources and packet reassembly at the destinations.

To be concise, in the rest of this paper we simply refer to time slot  $t$  as “slot  $t$ .” Since there can be at most one packet from any link at any slot  $t$  (by assumption (v)), we characterize a link by its link state and say that a link is in state 1 (resp.,

state 0) at slot  $t$  if there is a packet (resp., there is no packet) from that link at slot  $t$ .

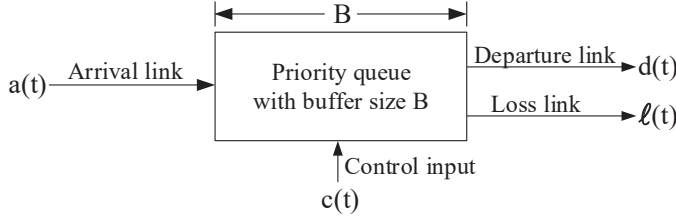


Fig. 3. A priority queue with buffer size  $B$ .

A priority queue with buffer size  $B$  is a network element with one arrival link, one departure link, one loss link, and one control input (see Figure 3). We denote  $a(t)$ ,  $d(t)$ , and  $\ell(t)$  as the link states of the arrival link, the departure link, and the loss link, respectively, at slot  $t$ . We denote  $c(t) = 1$  (resp.,  $c(t) = 0$ ) if there is a departure request (resp., there is no departure request) from the controller at slot  $t$ . We also denote  $q(t)$  as the number of packets stored in the buffer of the priority queue at slot  $t$ .

At each slot  $t$ , each of the packets in the priority queue at slot  $t$ , including the packets buffered in the queue at slot  $t-1$  and the packets from the arrival link at slot  $t$ , is assigned a *distinct* priority. The priority is assigned in such a way that the priority of the arrival packet (if any) at slot  $t$  can be *arbitrarily* assigned and the *relative* priority order between any two packets (if any) buffered in the queue at slot  $t-1$  remains unchanged at slot  $t$ .

We define a priority queue with buffer size  $B$  by the following three properties:

(P1) *Nonidling and priority departure*: If there is a departure request from the controller and there are packets in the queue at slot  $t$ , i.e.,  $c(t) = 1$  and  $q(t-1) + a(t) \geq 1$ , then there is a departure packet at slot  $t$ , and the departure packet at slot  $t$  is the packet in the queue at slot  $t$  with the *highest* priority. Otherwise, there is no departure packet at slot  $t$ .

(P2) *Maximum buffer usage and priority loss*: If there is a buffer overflow at slot  $t$ , i.e.,  $c(t) = 0$ ,  $q(t-1) = B$ , and  $a(t) = 1$ , then there is a loss packet at slot  $t$ , and the loss packet at slot  $t$  is the packet in the queue at slot  $t$  with the *lowest* priority. Otherwise, there is no loss packet at slot  $t$ .

(P3) *Flow conservation*: Packets in the queue at slot  $t$  are either buffered in the queue at slot  $t$  or transmitted through the departure link or the loss link at slot  $t$ .

### B. The Constructions of Optical Priority Queues in [39]

We recall the following two results from [39]. The first result tells us, for given  $1 \leq s \leq k-1$  and  $m_1, m_2, \dots, m_k \geq 1$ , how to choose the parameters  $n_1, n_2, \dots, n_k$ ,  $B_1, B_2, \dots, B_k$ , and  $|\Psi_1|, |\Psi_2|, \dots, |\Psi_k|$  so that the feedback system in Figure 2 can be operated as an optical priority queue under the priority-based routing policy in [39] (we note that the parameters  $s$  and  $|\Psi_1|, |\Psi_2|, \dots, |\Psi_k|$  are used in the priority-based routing policy in [39]).

**Theorem 1** [39, Theorem 1] Suppose that  $1 \leq s \leq k-1$  and  $m_1, m_2, \dots, m_k \geq 1$ . Then the feedback system in Figure 2 can be operated as an optical priority queue with buffer size  $U_k = \sum_{i=1}^k |\Psi_i|$  under the priority-based routing policy in [39] if the parameters  $n_1, n_2, \dots, n_k$ ,  $B_1, B_2, \dots, B_k$ , and  $|\Psi_1|, |\Psi_2|, \dots, |\Psi_k|$  satisfy the following conditions (A1)–(A3):

(A1) The condition for  $n_1, n_2, \dots, n_k$ :

$$n_i \geq \lceil (\sum_{j=j_1}^{j_2} m_j + 1) / m_i \rceil, \quad (1)$$

where  $j_1$  and  $j_2$  are given as follows (note that  $j_1$  and  $j_2$  depend on  $i$ ): If  $k$  is even, say  $k = 2\ell$ , then

$$\begin{cases} j_1 = \max\{i-1, 1\}, j_2 = \min\{i+s, \ell+1\}, & \text{if } 1 \leq i \leq \ell, \\ j_1 = \max\{i-s, \ell\}, j_2 = \min\{i+1, k\}, & \text{if } \ell+1 \leq i \leq k. \end{cases} \quad (2)$$

On the other hand, if  $k$  is odd, say  $k = 2\ell - 1$ , then

$$\begin{cases} j_1 = \max\{i-1, 1\} \text{ and } j_2 = \min\{i+s, \ell\}, & \text{if } 1 \leq i \leq \ell-1, \\ j_1 = i-1 \text{ and } j_2 = i+1, & \text{if } i = \ell, \\ j_1 = \max\{i-s, \ell\} \text{ and } j_2 = \min\{i+1, k\}, & \text{if } \ell+1 \leq i \leq k. \end{cases} \quad (3)$$

(A2) The condition for  $B_1, B_2, \dots, B_k$ :

$$1 \leq B_i \leq \begin{cases} U_{i-1} + 1, & \text{if } 1 \leq i \leq s+1, \\ U_{i-1} - U_{i-s-1}, & \text{if } s+2 \leq i \leq k, \end{cases} \quad (4)$$

and

$$1 \leq B_i \leq \begin{cases} U_{i+s} - U_i, & \text{if } 1 \leq i \leq k-s-1, \\ U_k - U_i + 1, & \text{if } k-s \leq i \leq k, \end{cases} \quad (5)$$

where  $U_i = \sum_{j=1}^i |\Psi_j|$  for  $i = 1, 2, \dots, k$ . Note that we have  $B_1 = B_k = 1$ .

(A3) The condition for  $|\Psi_1|, |\Psi_2|, \dots, |\Psi_k|$ :

$$1 \leq |\Psi_i| \leq (m_i - 1)B_i + 1 \text{ for } 1 \leq i \leq k, \quad (6)$$

and

$$|\Psi_i| \geq \begin{cases} B_{i-1}, & \text{if } 2 \leq i \leq \lceil k/2 \rceil, \\ B_{i+1}, & \text{if } \lceil k/2 \rceil + 1 \leq i \leq k-1. \end{cases} \quad (7)$$

The second result gives us the parameters  $n_1, n_2, \dots, n_k$ ,  $B_1, B_2, \dots, B_k$ , and  $|\Psi_1|, |\Psi_2|, \dots, |\Psi_k|$  for the *optimal* construction that achieves minimum construction complexity and maximum buffer size among the constructions in Theorem 1 for the scenario that  $m_i = m_{k-i+1} \geq 2$  for all  $1 \leq i \leq \lceil k/2 \rceil$ .

**Theorem 2** [39, Theorem 4] Suppose that  $1 \leq s \leq k-1$  and  $m_i = m_{k-i+1} \geq 2$  for all  $1 \leq i \leq \lceil k/2 \rceil$ . Then the parameters  $n_1, n_2, \dots, n_k$ ,  $B_1, B_2, \dots, B_k$ , and  $|\Psi_1|, |\Psi_2|, \dots, |\Psi_k|$  for the *optimal* construction that achieves minimum construction complexity and maximum buffer size among the constructions in Theorem 1 are given as follows:

(A1\*) The optimal choice for  $n_1, n_2, \dots, n_k$ :

$$n_i = \lceil (\sum_{j=j_1}^{j_2} m_j + 1) / m_i \rceil, \quad (8)$$

where  $j_1$  and  $j_2$  are given by (2) and (3) (note that  $j_1$  and  $j_2$  depend on  $i$ ), for  $1 \leq i \leq k$ .

(A2\*) The optimal choice for  $B_1, B_2, \dots, B_k$ : If  $s+1 \leq k \leq 2s+2$ , then  $B_1, B_2, \dots, B_k$  are recursively given by

$$B_i = B_{k-i+1} = \sum_{j=1}^{i-1} ((m_j - 1)B_j + 1) + 1 \quad \text{for } 1 \leq i \leq \lceil k/2 \rceil, \quad (9)$$

where we have adopted the convention that the sum is zero if the upper limit is smaller than the lower limit of a summation. On the other hand, if  $k \geq 2s+3$ , then  $B_1, B_2, \dots, B_k$  are recursively given by

$$B_i = B_{k-i+1} = \begin{cases} \sum_{j=1}^{i-1} ((m_j - 1)B_j + 1) + 1, & \text{if } 1 \leq i \leq s+1, \\ \sum_{j=i-s}^{i-1} ((m_j - 1)B_j + 1), & \text{if } s+2 \leq i \leq \lceil k/2 \rceil. \end{cases} \quad (10)$$

Note that we have  $B_1 = B_k = 1$ .

(A3\*) The optimal choice for  $|\Psi_1|, |\Psi_2|, \dots, |\Psi_k|$ :

$$|\Psi_i| = (m_i - 1)B_i + 1 \quad \text{for } 1 \leq i \leq k. \quad (11)$$

Furthermore, the maximum buffer size  $U_k$  achieved by the optimal construction is given by

$$U_k = \sum_{i=1}^k ((m_i - 1)B_i + 1). \quad (12)$$

### III. FAULT-TOLERANT CAPABILITY OF THE OPTIMAL CONSTRUCTIONS IN THEOREM 2

In this section, we consider the scenario that each group of FM1's in Figure 2 has the same number of FM1's, say,  $m_i = m$  for  $1 \leq i \leq k$ , and use Theorem 1 to show that the optimal construction given in Theorem 2 (with  $m_i = m$  for  $1 \leq i \leq k$ ) possesses fault-tolerant capability.

**Theorem 3** Suppose that  $1 \leq s \leq k-1$  and  $m_i = m \geq 2$  for  $1 \leq i \leq k$ . Then the optimal construction in Theorem 2 (with  $m_i = m$  for  $1 \leq i \leq k$ ) possesses fault-tolerant capability that can tolerate up to  $f$  malfunctioning FM1's, i.e., the feedback system in Figure 2 can still be operated as an optical priority queue but with a smaller buffer size after up to  $f$  FM1's fail to function properly, where  $f = \min_{1 \leq i \leq k} \lfloor (m-1)/(n_i-1) \rfloor$ .

**Proof.** Note that for the optimal construction in Theorem 2 (with  $m_i = m$  for  $1 \leq i \leq k$ ), the parameters  $n_1, n_2, \dots, n_k$  are given by (8),  $B_1, B_2, \dots, B_k$  are given by (9) and (10),  $|\Psi_1|, |\Psi_2|, \dots, |\Psi_k|$  are given by (11), and the maximum buffer size  $U_k$  achieved by the optimal construction is given by (12) (with  $m_i = m$  in (8)–(12) for  $1 \leq i \leq k$ ). Since the optimal construction in Theorem 2 is one of the constructions in Theorem 1, the parameters  $n_1, n_2, \dots, n_k, B_1, B_2, \dots, B_k$ , and  $|\Psi_1|, |\Psi_2|, \dots, |\Psi_k|$  satisfy the conditions (A1)–(A3).

Assume that there are  $f_i$  broken FM1's in the  $i^{\text{th}}$  group of FM1's in Figure 2 for  $1 \leq i \leq k$ , where  $\sum_{i=1}^k f_i \leq f$ . Let  $f'_i = \max\{f_i, f_{k-i+1}\}$  and let  $m'_i = m - f'_i$  for  $1 \leq i \leq k$ .

Then it is clear that  $f'_i = f'_{k-i+1} \leq \sum_{j=1}^k f_j \leq f$  and  $m'_i = m'_{k-i+1}$  for  $1 \leq i \leq \lceil k/2 \rceil$ . For  $1 \leq i \leq k$ , we have

$$\begin{aligned} m'_i &= m - f'_i \geq m - f \\ &\geq m - \lfloor (m-1)/(n_i-1) \rfloor = \lceil m - (m-1)/(n_i-1) \rceil \\ &\geq \lceil m - (m-1)/2 \rceil = \lceil (m+1)/2 \rceil \geq 2, \end{aligned} \quad (13)$$

where the first inequality follows from  $f'_i \leq f$ , the second inequality follows from  $f = \min_{1 \leq j \leq k} \lfloor (m-1)/(n_j-1) \rfloor \leq \lfloor (m-1)/(n_i-1) \rfloor$ , the third inequality follows from  $n_i \geq 3$  (this can be seen from (8), (2), and (3) (with  $m_i = m$  for  $1 \leq i \leq k$  in (8), (2), and (3)). and the fourth inequality follows from  $m \geq 2$ .

Let  $B'_1, B'_2, \dots, B'_k$  be given by (9) and (10) (with  $m_i$  in (9) and (10) replaced by  $m'_i$  for  $1 \leq i \leq k$ ), and let  $|\Psi'_1|, |\Psi'_2|, \dots, |\Psi'_k|$  be given by (11) (with  $m_i$  and  $B_i$  in (11) replaced by  $m'_i$  and  $B'_i$ , respectively, for  $1 \leq i \leq k$ ). In the following, we show that the parameters  $n_1, n_2, \dots, n_k, B'_1, B'_2, \dots, B'_k$ , and  $|\Psi'_1|, |\Psi'_2|, \dots, |\Psi'_k|$  satisfy the conditions (A1)–(A3) in Theorem 1 (with  $m_i$  replaced by  $m'_i$  for  $1 \leq i \leq k$ ), and it then follows from Theorem 1 that the feedback system in Figure 2 can still be operated as an optical priority queue by using the remaining functioning FM1's.

First we prove that the parameters  $n_1, n_2, \dots, n_k$  satisfy the condition (A1), i.e.,  $n_1, n_2, \dots, n_k$  satisfy (1) (with  $m_i$  in (1) replaced by  $m'_i$  for  $1 \leq i \leq k$ ) in the condition (A1). To see this, suppose that  $1 \leq i \leq k$ . Let  $j_1$  and  $j_2$  be given by (2) and (3) (note that  $j_1$  and  $j_2$  depend on  $i$ ). Then we have

$$\begin{aligned} &\sum_{j=j_1}^{j_2} m'_j + 1 \\ &= \sum_{j=j_1}^{j_2} (m - f'_j) + 1 = (j_2 - j_1 + 1)m - \sum_{j=j_1}^{j_2} f'_j + 1 \\ &= (j_2 - j_1 + 1)(m'_i + f'_i) - \sum_{j=j_1}^{j_2} f'_j + 1 \\ &\leq (n_i - 1)(m'_i + f) - f'_i + 1 \\ &\leq (n_i - 1)m'_i + (m - 1) - f'_i + 1 \\ &= (n_i - 1)m'_i + m'_i = n_i \cdot m'_i, \end{aligned} \quad (14)$$

where the first inequality follows from  $n_i = j_2 - j_1 + 2$  in (8) (with  $m_j = m$  in (8) for  $1 \leq j \leq k$ ),  $f'_i \leq f$ , and  $\sum_{j=j_1}^{j_2} f'_j \geq f'_i$  (as  $j_1 \leq i \leq j_2$ ), and the second inequality follows from  $f = \min_{1 \leq j \leq k} \lfloor (m-1)/(n_j-1) \rfloor \leq (m-1)/(n_i-1)$ . Thus, we obtain from (14) the desired result that  $n_i \geq \lceil (\sum_{j=j_1}^{j_2} m'_j + 1) / m'_i \rceil$ .

Since we know from (13) that  $m'_i = m'_{k-i+1} \geq 2$  for  $1 \leq i \leq \lceil k/2 \rceil$ , and the parameters  $B'_1, B'_2, \dots, B'_k$  and  $|\Psi'_1|, |\Psi'_2|, \dots, |\Psi'_k|$  are determined by  $m'_1, m'_2, \dots, m'_k$  in the same way that the parameters  $B_1, B_2, \dots, B_k$  and  $|\Psi_1|, |\Psi_2|, \dots, |\Psi_k|$  are determined by  $m_1, m_2, \dots, m_k$ , it then follows that the parameters  $B'_1, B'_2, \dots, B'_k$  and  $|\Psi'_1|, |\Psi'_2|, \dots, |\Psi'_k|$  satisfy the conditions (A2) and (A3) (just like the parameters  $B_1, B_2, \dots, B_k$  and  $|\Psi_1|, |\Psi_2|, \dots, |\Psi_k|$  satisfy the conditions (A2) and (A3)).

Finally, since  $m'_i = m - f'_i \leq m$  for  $1 \leq i \leq k$ , it is clear from (9) and (10) that  $B'_i \leq B_i$  for  $1 \leq i \leq k$ . As such, the  $n_i$  FM1's with buffer size  $B_i$  can be used as  $n_i$  FM1's with buffer size  $B'_i$  for  $1 \leq i \leq k$ . Therefore, we have from Theorem 1 that the feedback system in Figure 2 can still be

operated as an optical priority queue with buffer size  $U'_k = \sum_{i=1}^k ((m'_i - 1)B'_i + 1)$ . As we can see from  $m'_i \leq m$  and  $B'_i \leq B_i$  for  $1 \leq i \leq k$  that

$$\begin{aligned} U'_k &= \sum_{i=1}^k ((m'_i - 1)B'_i + 1) \\ &\leq \sum_{i=1}^k ((m - 1)B_i + 1) = U_k, \end{aligned}$$

we know that the buffer size  $U'_k$  achieved by using the remaining functioning FM1's is less than or equal to the buffer size  $U_k$  achieved when no FM1's are broken. ■

**Remark 4** (i) *The result in [39, Theorem 10] is a special case of our result in Theorem 3 when each FM1 in Figure 2 has the same number of arrival links.*

(ii) *The result in Theorem 3 can be used to provide quality of service (QoS). For example, to guarantee a fault-tolerant capability of  $f^*$  broken FM1's, the parameters  $m$  and  $n_1, n_2, \dots, n_k$  have to be chosen such that  $\min_{1 \leq i \leq k} \lfloor (m - 1)/(n_i - 1) \rfloor \geq f^*$ .*

#### IV. CONCLUSION

In this paper, we addressed the fault-tolerant capability of the optimal constructions of optical priority queues in [39] by using the feedback system in Figure 2. We considered the scenario that each group of FM1's has the same number of FM1's, and showed that the optimal constructions in [39] possess fault-tolerant capability which can be expressed in terms of the number of FM1's in each group and the numbers of arrival links of the FM1's in the  $k$  groups in Figure 2. Such a result can be used in the design of the constructions to provide QoS that guarantees a certain level of fault-tolerant capability.

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